PHYSICAL REVIEW D VOLUME 10, NUMBER 2 15 JULY 1974

Experimental consequences of objective local theories*

John F. Clauser
Department of Physics and Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

Michael A. Horne
Department of Physics, Stonehill College, North Easton, Massachusetts 02356

(Received 10 August 1973; revised manuscript received 8 April 1974)

A broad class of theories, called "objective local theories," is defined, motivation for considering these theories is given, and experimental consequences of the class are investigated. An extension of previous analyses by Bell and by Clauser *et al.* shows that predictions of objective local theories and of quantum mechanics differ, and that an experimental test of the entire family of objective local theories can be performed. The experimental requirements are given. Objective local theories satisfying a plausible but experimentally untestable supplementary assumption are shown to be incompatible with existing experimental data.

I. INTRODUCTION

Two papers by Bell have shown that the statistical predictions of quantum mechanics, for certain spatially separated yet correlated two-particle systems, are incompatible with a broad class of local theories. Bell's earlier paper considers the consequences of a physically reasonable locality condition within the domain of an ideal *Gedankenexperiment*.[1] He demonstrates that any theory which satisfies the locality condition must also be deterministic if certain quantum-mechanical predictions are valid for the idealized case. Bell's further analysis shows, however, that any deterministic local theories are necessarily incompatible with some other quantum-mechanical predictions for the *Gedankenexperiment*.

Upon examining the proof in Bell's earlier paper, one might conjecture that it is essentially the deterministic character of the class of theories that is incompatible with quantum mechanics.[2] That is, if the hypotheses assumed for the *Gedankenexperiment* were slightly relaxed so that determinism is no longer derivable,[3] then the incompatibility with the quantum-mechanical predictions will also be removed. This conjecture is incorrect. Bell shows in his second paper[4] that any stochastic theory satisfying the locality condition is also incompatible with quantum mechanics.

Inspired by Bell's first paper, Clauser *et al.*[5] showed that his analysis can be extended to cover realizable systems and that experimental tests of the broad class of local theories covered by these theorems can be performed. Although existing two-particle sources and/or analyzer-detector apparatuses appear insufficiently close to ideal for the desired experiment, Clauser *et al.* showed that, with a plausible but untestable supplementary assumption concerning detector efficiencies, deterministic local theories are incompatible with the quantum-mechanical predictions for a realizable experiment. Experimental results obtained by Freedman and Clauser[6] are in excellent agreement with the relevant quantum-mechanical predictions and thereby indicate that any deterministic local theory is untenable if the supplementary assumption is true. Clauser *et al.* in their original proposal for this experiment restricted their discussion to deterministic local theories. Following Bell's second paper, however, it was noted[7] that the experiment also indicates that any stochastic local theory is untenable if the same supplementary assumption concerning the detectors is made. However, the question of the experimental testability of stochastic local theories with either a weaker auxiliary assumption or with no auxiliary assumption has not previously been discussed.

The present paper extends the previous discussions of deterministic and stochastic local theories in several respects: (a) In preparation for the extension, we characterize explicitly a broad class of theories, which we designate as *objective local theories* (OLT), and discuss the fundamental premises which motivate them. Incompatibility of this class of theories with quantum mechanics has essentially been demonstrated by Bell,[4] but the result is in a form that is not practically experimentally testable. (b) We give a new incompatibility theorem that yields an experimentally testable result. (c) We show that, without an auxiliary assumption, neither the existing results of Freedman and Clauser nor any future refinement of their experiment employing more efficient detectors can provide a test of OLT because the angular correlation of the photon pairs is unsuitable. We note, however, that there do exist two-particle sources which are suitable for a test. (d) We state a supplementary assumption, weaker than that previously employed, and prove that it is sufficient to ensure the incompatibility of OLT and the experimental results of Freedman and Clauser. (e) We construct an explicit OLT model which reproduces the results of that experiment. We thereby prove that the Freedman-Clauser results constitute a refutation of only those OLT which satisfy our (or some similar) supplementary assumption.

II. OBJECTIVE LOCAL THEORIES

We will formulate and motivate objective local theories in the context of the experimental arrangement shown schematically in Fig. 1. A source of coincident two-particle emissions is viewed by two analyzer-detector assemblies 1 and 2. Each apparatus has an adjustable parameter; let a denote the value of the parameter at apparatus 1, and b that at apparatus 2. In Fig. 1, a and b are taken to be angles specifying the orientations of the analyzers, e.g., the axes of linear polarizers for photons, or the directions of the field gradients of Stern-Gerlach magnets for spin-$\frac{1}{2}$ particles. However, neither of these particular interpretations of the parameters a and b is essential for the discussion which follows; a and b may denote the values of any adjustable parameter at apparatus 1 and 2, respectively. Finally, in addition to an adjustable component and a detector, each apparatus may (and in practice does) contain various other components, such as additional filters to shield the detectors from unwanted radiations, etc. Since we require that these additional apparatus components remain in place during the experiment, we will ignore them in the discussion. Similarly, we ignore and assume constant any other macroscopic variables, such as those describing the source-apparatus geometry.

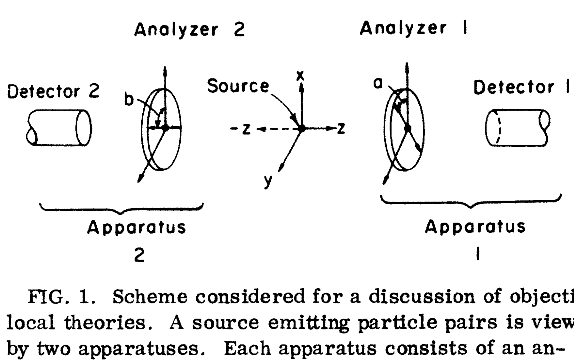

FIG. 1. Scheme considered for a discussion of objective local theories. A source emitting particle pairs is viewed by two apparatuses. Each apparatus consists of an analyzer and an associated detector. The analyzers have parameters, a and b respectively, which are externally adjustable. In the above example, a and b represent the angles between the analyzer axes and a fixed reference axis.

During a period of time, while the adjustable parameters have the values a and b, the source emits, say, N of the two-particle systems of interest.[8] For this period, denote by $N_1(a)$ and $N_2(b)$ the number of counts at detectors 1 and 2, respectively, and by $N_{12}(a, b)$ the number of simultaneous counts from the two detectors (coincident counts).[9] If N is sufficiently large, then the ensemble probabilities of these results are

$$\begin{aligned} p_1(a) &= N_1(a)/N\,, \\ p_2(b) &= N_2(b)/N\,, \\ p_{12}(a, b) &= N_{12}(a, b)/N\,. \end{aligned} \tag{1}$$

Consider one of the two-component emissions from the source. Physical theories, classical, quantum-mechanical, and presumably more general ones as well, characterize a physical system with a *state*. Moreover, during the system's existence, its state in general evolves. Consider the state specification of the above system at a time intermediate between its emission and its impingement on either apparatus.[10] Denote this state by λ. Note that we do not necessarily make a commitment to the completeness of this state specification, i.e., it may or may not describe the ultimate essence of the system at the chosen time. But neither do we make any restriction on the possible complexity of λ, nor do we assume it has any special characteristics; in short, we assume no model. As the state described initially by λ subsequently evolves, it may or may not trigger a count at apparatus 1, and similarly it may or may not do so at apparatus 2. The initial state λ, if it serves the same role as in existing theories, will suffice to determine *at least* the probabilities of these events.[11] Let the probabilities of a count being triggered at apparatus 1 and 2 be $p_1(\lambda, a)$ and $p_2(\lambda, b)$, respectively, and let $p_{12}(\lambda, a, b)$ be the probability that both counts are triggered.[12]

Since, in general, every system in the ensemble emitted by the source may not have the same initial state, we allow a mixture of states. Let $\rho(\lambda)$ be the normalized probability density characterizing the ensemble of emissions.[13] In terms of the quantities just defined, the ensemble probabilities given in Eqs. (1) are

$$\begin{aligned} p_1(a) &= \int_\Gamma d\lambda\, \rho(\lambda) p_1(\lambda, a)\,, \\ p_2(b) &= \int_\Gamma d\lambda\, \rho(\lambda) p_2(\lambda, b)\,, \\ p_{12}(a, b) &= \int_\Gamma d\lambda\, \rho(\lambda) p_{12}(\lambda, a, b)\,, \end{aligned} \tag{2}$$

where Γ is the space of the states λ. The formulation (2) is quite general. Nothing so far has been assumed that is not satisfied by quantum mechanics. It suffices in Eqs. (2) to let λ define a quantum state and let $\rho(\lambda)$ define a distribution over quantum states implicit in a mixture.

Hereafter, we focus our attention on a special case of formulation (2) in which

$$p_{12}(\lambda, a, b) = p_1(\lambda, a) p_2(\lambda, b) . \qquad (2')$$

What considerations motivate this factored form? Clearly, if each source emission consists of two well-localized subsystems, e.g., a pair of objective particles, and there is no action at a distance, then the factored form is a reasonable locality condition. More generally, the factored form is a natural expression of a field-theoretical point of view, which in turn is an extrapolation from the common-sense view that there is no action at distance.

Fields propagate and can impinge upon different localized objects, among them pieces of apparatus. Since λ describes the field initially,[14] and the parameter a is associated with only one apparatus, it is reasonable that there is a well-defined probability $p_1(\lambda, a)$ that that apparatus will be triggered. The impingement of the field on this apparatus will naturally modify the field. However, if the events marking the action of the field on two pieces of apparatus (triggering or not in each case) have spacelike separation, then the reaction upon the field due to impingement on the first apparatus will not have time to affect the part of the field impinging the second apparatus, and conversely. Hence, the probability p_2 that the second apparatus will be triggered will not depend on whether or not the first has been triggered, or upon the choice of the parameter a at the first apparatus, or even upon the presence of the first apparatus. A similar assertion of independence holds for p_1, whence (2′) follows. In view of the foregoing motivation of Eq. (2′), we call any theory in which it holds an objective local theory.[15]

It should be recalled that the wave function was introduced by Schrödinger from a field-theoretical point of view, and most physicists have continued to think of it field-theoretically, whatever their precautions when they are on guard about the character of quantum mechanics. Hence, we conjecture that Eq. (2′) is implicit in the thinking of many physicists.[16] Whether or not this is correct, it is apparent that quantum mechanics is not of the form (2′). For many two-quanta sources, every emission is described by the same pure state, suggesting that this quantum state be identified with one value of λ, say λ', and that $\rho(\lambda) = \delta(\lambda - \lambda')$ (no mixture). However, when the two-particle quantum state is not a simple product of single-particle states but is instead a superposition of such products, the quantum-mechanical probability p_{12} does not in general admit the factorization (2′). The only alternative for saving Eq. (2′) is to identify the quantum-mechanical pure state with $\rho(\lambda)$, i.e., with a mixture of other states λ.[17] But, as we shall see, this identification is impossible for some quantum-mechanical pure states.

III. EXPERIMENTAL CONSEQUENCES

Measurement of the probabilities (1) requires not only the numbers $N_1(a)$, $N_2(b)$, and $N_{12}(a, b)$, which are directly observed quantities in a counting experiment, but also the number N, which is generally unobservable in an experiment counting microsystems. In practice, the number of emissions N occurring during any time period is usually deduced from the counting data for that period, since intervening counters will in practice depolarize (if not destroy) the systems. But this deduction always depends upon the currently accepted theoretical description of the whole phenomenon—the source, the apparatus, and their interactions. That is, N is actually deduced from Eqs. (1) *themselves* with the p's supplied by the theory at hand. Clearly, any such method of determining N must not be employed in an experimental test of competing theories. Therefore, in this section, we derive a consequence of Eq. (2′) which is experimentally testable without N being known, and which contradicts the quantum-mechanical predictions.

Let a and a' be two orientations of analyzer 1, and let b and b' be two orientations of analyzer 2. The inequalities

$$\begin{aligned} &0 \le p_1(\lambda, a) \le 1 , \\ &0 \le p_1(\lambda, a') \le 1 , \\ &0 \le p_2(\lambda, b) \le 1 , \\ &0 \le p_2(\lambda, b') \le 1 \end{aligned} \qquad (3)$$

hold if the probabilities are sensible. These inequalities and the theorem in Appendix A give

$$-1 \le p_1(\lambda, a)p_2(\lambda, b) - p_1(\lambda, a)p_2(\lambda, b') + p_1(\lambda, a')p_2(\lambda, b) + p_1(\lambda, a')p_2(\lambda, b') - p_1(\lambda, a') - p_2(\lambda, b) \le 0$$

for each λ. Multiplication by $\rho(\lambda)$ and integration over λ gives

$$-1 \le p_{12}(a, b) - p_{12}(a, b') + p_{12}(a', b) + p_{12}(a', b') - p_1(a') - p_2(b) \le 0 \qquad (4)$$

as a necessary constraint on the statistical predictions of any OLT. If, for some reason such as rotational invariance, it is found experimentally that $p_1(a)$ and $p_2(b)$ are constant, and that $p_{12}(a, b) = p_{12}(\phi)$ holds, where $\phi = |b-a|$ is the angle between the analyzer axes, then (4) becomes

$$-1 \leqslant 3p_{12}(\phi) - p_{12}(3\phi) - p_1 - p_2 \leqslant 0 . \tag{4'}$$

Here, a, a', b, and b' have been chosen so that

$$|a-b| = |a'-b| = |a'-b| = \tfrac{1}{3}|a-b'| = \phi .$$

The upper limits in (4) and in (4′) are experimentally testable without N being known. Inequalities (4) and (4′) hold perfectly generally for any systems described by OLT. These are new results not previously presented elsewhere. The relationship between them and the previous inequalities of Bell is discussed in Appendix B.

IV. INCOMPATIBILITY WITH QUANTUM MECHANICS

We now present the conditions under which the predictions (4) and (4′) are incompatible with those of quantum mechanics. Consider an experiment with a configuration as described above whose quantum-mechanical predictions take the following form:

$$\begin{aligned} p_{12}(\phi) &= \tfrac{1}{4}\eta_1\eta_2 f_1 g[\epsilon_+^1 \epsilon_+^2 + \epsilon_-^1\epsilon_-^2 F \cos(n\phi)] ,\\ p_1 &= \tfrac{1}{2}\eta_1 f_1 \epsilon_+^1 ,\\ p_2 &= \tfrac{1}{2}\eta_2 f_2 \epsilon_+^2 . \end{aligned} \tag{5}$$

This general form is characteristic of many correlation phenomena, e.g., the spin-$\frac{1}{2}$–spin-$\frac{1}{2}$ correlation *Gedankenexperiment* introduced by Bohm[18] and used by Bell,[1,4] and the actual experiments of Freedman and Clauser,[6] Wu and Shaknov,[19] and Kasday *et al.*[20] Inserting the predictions (5) into (4′) and selecting the optimum value $\phi = 45°/n$, one finds that the condition for a violation of the upper bound is

$$\eta g \epsilon_+ [\sqrt{2}\,(\epsilon_-/\epsilon_+)^2 F + 1] \geqslant 2 . \tag{6}$$

Here for simplicity we have taken $\eta_1 = \eta_2 \equiv \eta$, $f_1 = f_2$, $\epsilon_+^1 = \epsilon_+^2 \equiv \epsilon_+$, and $\epsilon_-^1 = \epsilon_-^2 \equiv \epsilon_-$. Thus, a correlation experiment with values in the domain specified by (6) is capable of distinguishing between OLT and quantum theory. Although experiments are possible for which this is the case, there are at present no existing experimental results satisfying (6), and thus none which are in violation of (4′) and/or (4).

Consider next the specific example of the photon pairs emitted by a $J=0 \rightarrow J=1 \rightarrow J=0$ atomic cascade. A source of such pairs was used in the experiment of Freedman and Clauser. Figure 2 is a diagram of their experiment; it is clearly of the configuration discussed in the previous section. For this arrangement, the predictions are given by Eqs. (5) with $n=2$ and the following identifications[5]: η_i is the quantum efficiency of detector i $(i=1, 2)$, and

$$\epsilon_+^i \equiv \epsilon_M^i + \epsilon_m^i \text{ and } \epsilon_-^i \equiv \epsilon_M^i - \epsilon_m^i .$$

Here ϵ_M^i is the efficiency of polarizer i for light polarized parallel to the polarizer axis, and ϵ_m^i is the efficiency for light polarized perpendicular to it. The function $f_1 = f_2 = f(\theta)$ is the probability that the $J=0 \rightarrow J=1$ $(J=1 \rightarrow J=0)$ emission enters apparatus 1 (apparatus 2),

$$f(\theta) = \tfrac{1}{2}(1 - \cos\theta) , \tag{7}$$

with θ being the half-angle of the cones subtended by each detector aperture. The function $g = g(\theta)$ is the conditional probability, or angular correlation factor, that if the $J=0 \rightarrow J=1$ emission enters apparatus 1 then the $J=1 \rightarrow J=0$ emission will enter apparatus 2,

$$g(\theta) = \frac{3}{8} \frac{[G_2(\theta)]^2 + \frac{1}{2}[G_3(\theta)]^2}{1-\cos\theta} . \tag{8}$$

Finally,

$$F = F(\theta) = \frac{2[G_1(\theta)]^2}{[G_2(\theta)]^2 + \frac{1}{2}[G_3(\theta)]^2} \tag{9}$$

reflects a depolarization effect due to noncollinearity of the two photons and approaches unity for infinitesimal detector apertures $(\theta \rightarrow 0)$. The functions G_1, G_2, and G_3 are given in Ref. 5.

Inserting Eqs. (8) and (9) into (6) one finds that because of the relatively small value of $g(\theta)$ condition (6) is *not* satisfied for any value of the detector half-angle θ, even if the analyzer and detector efficiencies are ideal $(\eta = \epsilon_+ = \epsilon_- = 1)$.[21] Therefore, for cascade-photon experiments, the quantum-mechanical predictions are compatible with (4) even in the domain of ideal apparatus. The

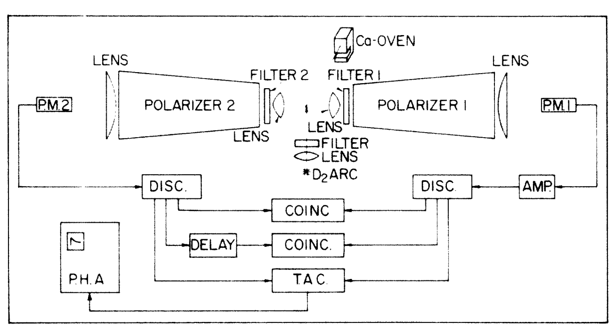

FIG. 2. Schematic diagram of apparatus and associated electronics of the experiment by Freedman and Clauser. Scalers (not shown) monitored the outputs of the discriminators and coincidence circuits. (Figure after Freedman and Clauser.)

insufficient magnitude of the angular correlation factor $g(\theta)$ is a consequence of the fact that an atomic cascade is a three-body decay, the atom being the third body.

However, for correlated particles produced in certain two-body decays, the quantum-mechanical predictions violate (4). The annihilation of ground-state positronium into two γ rays or the dissociation of a spin-0 or a spin-1 molecule into two spin-$\frac{1}{2}$ particles produce correlations of the form (5). Since these are two-body decays, $g(\theta)=1$ holds even for small θ (provided the center-of-mass velocity of the decaying object is sufficiently small). Even with $g=1$, inequality (6) imposes rather stringent conditions on the efficiencies of the analyzers and detectors. But there appears to be no *a priori* reason why such conditions cannot be achieved in practice. This question will be the subject of future work.

V. CONSEQUENCES WITH A SUPPLEMENTARY ASSUMPTION

Until a correlation experiment employing highly efficient analyzers and detectors is performed on two-body decays,[22] it is desirable to exhibit a physically plausible supplementary assumption which makes the existing cascade-photon experiment applicable as a test of OLT. In this section we state an assumption, weaker than the one previously presented by Clauser *et al.*, and prove that it is sufficient to make OLT incompatible with existing experimental results. In the next section we prove, with an explicit OLT model, that this or some other supplementary assumption is necessary for such an application.

The assumption is that, for every emission λ, the probability of a count with a polarizer in place is less than or equal to the probability with the polarizer removed. Let ∞ denote the absence of the polarizer, and let $p_1(\lambda,\infty)$ denote the probability of a count from detector 1 when the polarizer is absent and the emission is λ. A similar probability $p_2(\lambda,\infty)$ may be defined for apparatus 2. Thus, our assumption is

$$\begin{aligned} &0 \le p_1(\lambda, a) \le p_1(\lambda,\infty) \le 1 \ , \\ &0 \le p_2(\lambda, b) \le p_2(\lambda,\infty) \le 1 \ , \end{aligned} \qquad (10)$$

for every λ, and for all values of a and b. We call this the *no-enhancement assumption*.[23] We now make an argument analogous to that which led from (3) to (4). Inequalities (10) and the theorem of Appendix A yield immediately the result

$$-p_{12}(\infty,\infty) \le p_{12}(a,b) - p_{12}(a,b') + p_{12}(a',b) + p_{12}(a',b') - p_{12}(a',\infty) - p_{12}(\infty,b) \le 0 \ , \qquad (11)$$

where

$$p_{12}(x,y) \equiv \int_\Gamma d\lambda\, \rho(\lambda) p_1(\lambda,x) p_2(\lambda,y)$$

for all x and y. With the same conditions used in writing (4′), (11) becomes

$$p_{12}(\infty,\infty) \le 3p_{12}(\phi) - p_{12}(3\phi) - p_{12}(a',\infty) - p_{12}(\infty,b) \le 0 \ . \qquad (11')$$

Note that all terms in (11) are joint probabilities for coincident counts at the two detectors. Inequality (4), in contrast, contains the two terms p_1 and p_2 which are probabilities of a count at a single detector. The upper limit of (11), or (11′), is identical to the previous result of Clauser *et al.*, but their derivation was restricted to deterministic local theories and employed an auxiliary assumption stronger than the no-enhancement assumption.

The quantum-mechanical prediction for $p_{12}(\phi)$ in the cascade-photon experiment was given in Eqs. (5) and (7)–(9). The predictions for the other joint probabilities occurring in (11) are[5]

$$\begin{aligned} p_{12}(a',\infty) &= \tfrac{1}{2}\eta_1\eta_2 f(\theta) g(\theta)\epsilon_+^1 \ , \\ p_{12}(\infty,b) &= \tfrac{1}{2}\eta_1\eta_2 f(\theta) g(\theta)\epsilon_+^2 \ , \\ p_{12}(\infty,\infty) &= \eta_1\eta_2 f(\theta) g(\theta) \ . \end{aligned} \qquad (12)$$

These predictions violate the upper bound in (11′) provided

$$\epsilon_+[\sqrt{2}\,(\epsilon_-/\epsilon_+)^2 F(\theta)+1] \ge 2 \qquad (13)$$

holds. As before, we have used the optimum value $\phi = 22\frac{1}{2}^\circ$ and set $\epsilon_+^1 = \epsilon_+^2 \equiv \epsilon_+$ and $\epsilon_-^1 = \epsilon_-^2 \equiv \epsilon_-$. Note that neither the angular correlation factor nor the detector efficiencies appear in (13). The apparatus of Freedman and Clauser satisfied (13), and the experimental results, which confirmed the quantum-mechanical predictions, substantially violated the upper bound of (11′). Consequently, in view of the theorem just given, any OLT that do not admit enhancement are untenable.[24]

VI. NECESSITY OF A SUPPLEMENTARY ASSUMPTION

To prove that an auxiliary assumption is necessary if the cascade-photon experiment is to refute OLT, we exhibit an explicit OLT model which reproduces the results of that experiment. Since the experimental results are in agreement with the quantum-mechanical predictions, it suffices to construct a model which reproduces these predictions. For simplicity, however, we exhibit the model only for the ideal case in which the detectors subtend infinitesimal solid angles $[\theta \to 0$ and $F(\theta) \to 1]$. The extension of the model to the finite solid angles of the actual experiment $[\theta = 30°$ and $F(\theta) = 0.99]$ introduces nothing new, and the additional complexity obscures the point. With these simplifications, the predictions to reproduce are

$$\begin{aligned} p_{12}(\phi)/p_{12}(\infty,\infty) &= \tfrac{1}{4}(\epsilon_+^1\epsilon_+^2 + \epsilon_-^1\epsilon_-^2\cos 2\phi)\,, \\ p_{12}(a,\infty)/p_{12}(\infty,\infty) &= \tfrac{1}{2}\epsilon_+^1\,, \\ p_{12}(\infty,b)/p_{12}(\infty,\infty) &= \tfrac{1}{2}\epsilon_+^2\,. \end{aligned} \tag{14}$$

Only ratios could be measured, since, with N unknown, the actual values of the p's were experimentally inaccessible.

The model is as follows: Each emission pair consists of two particles, such that particle 1 travels along the $+z$ axis to apparatus 1, and particle 2 travels along the $-z$ axis to apparatus 2. Both members of the pair possess a common state variable λ which is simply an azimuthal angle; that is, it specifies a direction perpendicular to the flight axis from some reference axis (see Fig. 3). The ensemble of emitted pairs is characterized by a normalized isotropic density

$$\rho(\lambda)d\lambda = \frac{d\lambda}{2\pi}\,, \quad 0 \le \lambda \le 2\pi\,. \tag{15}$$

From the same reference axis, we specify the orientations of polarizers 1 and 2 by the angles a and b. With the polarizer removed, the probability of a count at each detector is a constant, independent of λ:

$$p_1(\lambda,\infty) = c_1\,, \tag{16}$$

$$p_2(\lambda,\infty) = c_2\,. \tag{17}$$

The probability of a count at detector 1, given an emission λ and the setting a of an inserted polarizer, is

$$p_1(\lambda, a) = \tfrac{1}{2}c_1[\epsilon_+^1 + \epsilon_-^1\cos 2(\lambda - a)]\,. \tag{18}$$

At detector 2, the probability is

$$p_2(\lambda, b) = \tfrac{1}{2}c_2\pi\epsilon_+^2/\delta \tag{19}$$

for $b - \frac{1}{2}\delta \le \lambda \le b + \frac{1}{2}\delta$ and $\pi + b - \frac{1}{2}\delta \le \lambda \le \pi + b + \frac{1}{2}\delta$; $p_2(\lambda, b)$ is zero otherwise. Here δ is a function of the ratio $\epsilon_-^2/\epsilon_+^2$, and is defined by the relation

$$(\sin\delta)/\delta = \epsilon_-^2/\epsilon_+^2\,. \tag{20}$$

Clearly, this model is an OLT provided the probabilities (16) through (19) are less than unity. Evaluation of the ensemble probabilities yields

$$\begin{aligned} p_{12}(a,b) &= \tfrac{1}{4}c_1c_2[\epsilon_+^1\epsilon_+^2 + \epsilon_-^1\epsilon_-^2\cos 2(a-b)]\,, \\ p_{12}(a,\infty) &= \tfrac{1}{2}c_1c_2\epsilon_+^1\,, \\ p_{12}(\infty,b) &= \tfrac{1}{2}c_1c_2\epsilon_+^2\,, \\ p_{12}(\infty,\infty) &= c_1c_2\,, \end{aligned}$$

which agree with the quantum-mechanical ratios (14).

Note that for some values of the polarizer parameters ϵ_+^i and $\epsilon_-^i/\epsilon_+^i$ the model is enhancement-free. Since the function $p_1(\lambda, a)$ is less than $p_1(\lambda,\infty)$ for all physically sensible values of ϵ_+ and ϵ_-, it is enhancement-free in general. To see this note that, since $0 \le \epsilon_m \le \epsilon_M \le 1$ holds for any polarizer, it follows that $0 \le \epsilon_-^i/\epsilon_+^i \le 1$ and $\epsilon_+^i \le 2[(\epsilon_-^i/\epsilon_+^i)+1]^{-1}$ also hold. However, $p_2(\lambda, b)$ is enhancement-free if and only if

$$\epsilon_+^2 \le 2\delta/\pi\,. \tag{21}$$

The values $\epsilon_+^2 \approx 1.00$ and $\epsilon_-^2/\epsilon_+^2 \approx 0.94$ of the Freedman-Clauser apparatus do not satisfy (21), and therefore, as expected from the theorem of the previous section, the model requires enhancement to reproduce the experimental results. Note finally that, even with enhancement, $p_2(\lambda, b)$ remains sensible provided the constant c_2 of the model is sufficiently small. The value of $p_2(\lambda, b)$ is sensible for all λ and b provided $c_2 \le 2\delta(\pi\epsilon_+^2)^{-1}$. For the actual values of the experiment this condition is $c_2 \le 0.38$.

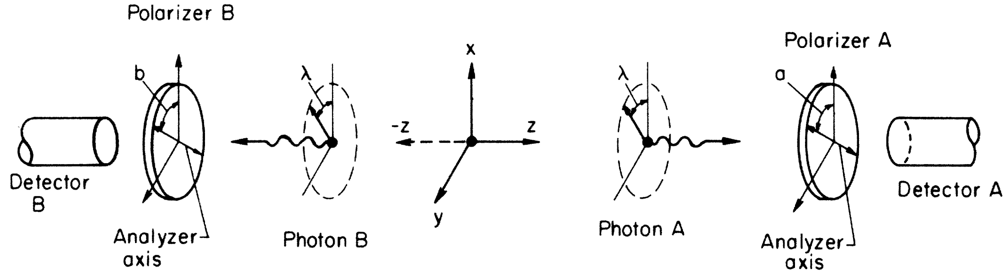

FIG. 3. Coordinate system for OLT model. Photon particles A and B carry the same azimuthal direction λ, which, along with the analyzer orientation a or b, determines the probability of a count at the associated detector.

VII. CONCLUSIONS

Physicists have consistently attempted to model microscopic and macroscopic phenomena in terms of objective entities, preferably with some definable structure. The present paper has addressed the question of whether or not the existing formalism of quantum mechanics can be recast or perhaps reinterpreted in a manner which restores the objectivity of nature, and thus allows such models (deterministic or not) to be made. We have found that it is not possible to do so in a natural way, consistent with locality, without an observable change of the experimental predictions.[26]

ACKNOWLEDGMENT

The authors gratefully acknowledge many valuable discussions with A. Shimony.

APPENDIX A: TWO INEQUALITIES

We prove the following theorem: Given six numbers x_1, x_2, y_1, y_2, X, and Y such that

$$\begin{aligned} &0 \leq x_1 \leq X,\\ &0 \leq x_2 \leq X,\\ &0 \leq y_1 \leq Y,\\ &0 \leq y_2 \leq Y, \end{aligned} \tag{A1}$$

then the function $U = x_1y_1 - x_1y_2 + x_2y_1 + x_2y_2 - Yx_2 - Xy_1$ is constrained by the inequalities

$$-XY \leq U \leq 0 . \tag{A2}$$

To establish the upper bound, consider two cases. First assume that $x_1 \geq x_2$ and rewrite (A2),

$$U = (x_1 - X)y_1 + (y_1 - Y)x_2 + (x_2 - x_1)y_2 .$$

We have thus assumed the last term to be nonpositive. Inequalities (A1) require the first two terms to be nonpositive, and the validity of the upper bound is demonstrated for this case. Next assume the other alternative, i.e., that $x_1 < x_2$, and use this assumption to bound U, thus:

$$\begin{aligned} U &= x_1(y_1 - y_2) + (x_2 - X)y_1 + x_2(y_2 - Y)\\ &\leq x_1(y_1 - y_2) + (x_2 - X)y_1 + x_1(y_2 - Y)\\ &= (x_2 - X)y_1 - x_1(y_1 - Y) \leq 0 . \end{aligned}$$

Thus, the upper bound is established in general.

The proof of the lower bound follows from a consideration of three cases. First, assume $x_2 \geq x_1$. The validity of the lower bound is apparent by inspection when written in the form

$$U + XY = (X - x_2)(Y - y_1) + x_1y_1 + (x_2 - x_1)y_2 ,$$

since (A1) requires all three terms to be nonnegative. Similarly for the case $y_1 \geq y_2$, inspection reveals the correctness of the lower bound when written

$$U + XY = (X - x_2)(Y - y_1) + x_2y_2 + x_1(y_1 - y_2) .$$

Finally, suppose neither of the two previous cases holds; that is, $x_2 < x_1$ *and* $y_1 < y_2$. Then write

$$\begin{aligned} U + XY &= (X - x_2)(Y - y_1)\\ &\quad - (x_1 - x_2)(y_2 - y_1) + x_2y_1 . \end{aligned}$$

The sum of the first two terms in non-negative since now $(X - x_2) \geq (x_1 - x_2) > 0$ and $(Y - y_1) \geq (y_2 - y_1) > 0$. By (A1) the final term is also nonnegative. Q.E.D.

APPENDIX B: INEQUALITIES (4) AND BELL'S THEOREM

Inequalities (4) assumed the arrangement of Fig. 1, where each apparatus consists of an analyzer with a single-channel output followed by a photomultiplier. Bell[4] considers a two-channel apparatus, e.g., a birefringent polarizing crystal followed by two photomultipliers, one photomuliplier monitoring the ordinary ray emerging from the crystal and the other monitoring the extraordinary ray.[27] Let $p_1^+(\lambda, a)$ denote the probability of a count in the ordinary channel of apparatus 1, and $p_1^-(\lambda, a)$ denote the probability of a count in the extraordinary channel, for the orientation a of the analyzer and initial state λ of the emission. Let $p_2^+(\lambda, b)$ and $p_2^-(\lambda, b)$ be similarly defined for apparatus 2. Bell considers the correlation function defined by[28]

$$\begin{aligned} P(a, b) &\equiv \int_\Gamma d\lambda\, \rho(\lambda)[p_1^+(\lambda, a) - p_1^-(\lambda, a)]\\ &\qquad \times [p_2^+(\lambda, b) - p_2^-(\lambda, b)]\\ &= p_{12}^{++}(a, b) - p_{12}^{+-}(a, b)\\ &\qquad - p_{12}^{-+}(a, b) + p_{12}^{--}(a, b) ,\\ p_{12}^{jk}(a, b) &\equiv \int_\Gamma d\lambda\, \rho(\lambda) p_1^j(\lambda, a) p_2^k(\lambda, b) ,\\ &\qquad \text{for } j, k = \pm 1 . \end{aligned} \tag{B1}$$

Using the fact that each square bracket in (B1) is bounded by ± 1, he proves that P is constrained by the inequalities

$$-2 \leq P(a, b) - P(a, b') + P(a', b) + P(a', b') \leq 2 . \tag{B2}$$

Note that for a direct experimental test of (B2) the number of emissions N must be known, and that in actual practice this probably cannot be found without either destroying or at least depolarizing the particles. [If N can be found, (B2) suffices for an experimental test.] Here we wish to show the relations between (4) and (B2).

First, (B2) is a corollary of (4). In an experiment employing two detectors (+ and −) behind each double-channel analyzer, inequalities (4) are still applicable and provide four sets of inequalities,

$$-1 \leq p_{12}^{jk}(a,b) - p_{12}^{jk}(a,b') + p_{12}^{jk}(a',b) + p_{12}^{jk}(a',b') - p_1^j(a') - p_2^k(b) \leq 0\ , \tag{B3}$$

where $j=\pm 1$ and $k=\pm 1$ indicate which detectors are considered. Multiplying the inequalities for which $j \neq k$ by −1 and adding these to the inequalities for which $j=k$, we obtain (B2).

Second, with Bell's formulation, (4) is not an immediate corollary of (B2). For a given analyzer setting a and emission λ, there are three possible results at apparatus 1: a count in the + detector, a count in the − detector, or no count in either detector. Let $p_1^0(\lambda, a)$ denote the probability of no count. Clearly

$$p_1^+(\lambda,a) + p_1^-(\lambda,a) + p_1^0(\lambda,a) = 1$$

holds, which implies

$$p_1^+(\lambda,a) - p_1^-(\lambda,a) = 2p_1^+(\lambda,a) + p_1^0(\lambda,a) - 1\ . \tag{B4}$$

With (B4), and a similar expression for apparatus 2, the expression (B1) defining the correlation function becomes

$$\begin{aligned} P(a,b) = {} & 4p_{12}^{++}(a,b) - 2p_1^+(a) - 2 - p_2^+(b) + 1 \\ & + 2p_{12}^{+0}(a,b) + 2p_{12}^{0+}(a,b) + p_{12}^{00}(a,b) \\ & - p_1^0(a) - p_2^0(b)\ . \end{aligned} \tag{B5}$$

Insertion of (B5), and similar expressions for $P(a',b)$, $P(a,b')$ and $P(a',b')$, into (B2) yields

$$-1 - Q \leq p_{12}^{++}(a,b) - p_{12}^{++}(a,b') + p_{12}^{++}(a',b) + p_{12}^{++}(a',b') - p_1^+(a') - p_2^+(b) \leq -Q\ , \tag{B6}$$

where

$$\begin{aligned} Q \equiv \tfrac{1}{2}[& p_{12}^{+0}(a,b) + p_{12}^{0+}(a,b) + \tfrac{1}{2}p_{12}^{00}(a,b) - p_{12}^{+0}(a,b') - p_{12}^{0+}(a,b') - \tfrac{1}{2}p_{12}^{00}(a,b') + p_{12}^{+0}(a',b) \\ & + p_{12}^{0+}(a',b) + \tfrac{1}{2}p_{12}^{00}(a',b) + p_{12}^{+0}(a',b') + p_{12}^{0+}(a',b') + \tfrac{1}{2}p_{12}^{00}(a',b') - p_1^0(a') - p_2^0(b)]\ . \end{aligned}$$

If we could establish that $Q \geq 0$, then the upper bound of (B6) would be identical to the experimentally useful upper bound of (4). No reason is immediately apparent that this is so, and unfortunately the terms p_{12}^{00}, p_1^0, and p_2^0 occurring in Q are unobservable, since they are probabilities of nothing happening. Thus auxiliary assumptions, and/or auxiliary experimentation as well, are necessary to test (B2) or, equivalently, (B6).

However, if Bell's formulation is modified *at the beginning,* his method of proof can be employed to obtain (4). Consider, instead of the correlation function (B1), the function

$$\begin{aligned} P'(a,b) & \equiv \int_\Gamma d\lambda\, \rho(\lambda)[2p_1^+(\lambda,a) - 1][2p_2^+(\lambda,b) - 1] \\ & = 4p_{12}^{++}(a,b) - 2p_1^+(a) - 2p_2^+(b) + 1\ . \end{aligned} \tag{B7}$$

Since each square bracket in P' is bounded by −1 and +1, Bell's method is applicable and yields

$$-2 \leq P'(a,b) - P'(a,b') + P'(a',b) + P(a',b') \leq 2. \tag{B8}$$

Using Eq. (B7), inequalities (B8) become

$$\begin{aligned} -1 \leq {} & p_{12}^{++}(a,b) - p_{12}^{++}(a,b') + p_{12}^{++}(a',b) \\ & + p_{12}^{++}(a',b') - p_1^+(a') - p_2^+(b) \leq 0\ , \end{aligned}$$

which, suppressing superscripts, is (4). In the context of a double-channel experiment, the three other sets of inequalities given in (B3) can be obtained in similar fashion.[29]

---

*Work by one of us (J.F.C.) was supported by the U. S. Atomic Energy Commission.

[1]J. S. Bell, Physics (N.Y.) 1, 195 (1965).

[2]K. Popper, in *Perspectives in Quantum Theory: Essays in Honor of Alfred Landé*, edited by W. Yourgrau and A. Van der Merwe (MIT Press, Cambridge, Mass., 1971), p. 182. For a reply to this conjecture, see J. S. Bell, Science 177, 880 (1972).

[3]Clauser *et al*. (Ref. 5) did relax the experimentally unattainable conditions of the *Gedankenexperiment*, but nevertheless retained determinism as a separate hypothesis.

[4]J. S. Bell, in *Foundations of Quantum Mechanics, Proceedings of the International School of Physics "Enrico*

Fermi," *Course XLIX,* edited by B. d'Espagnat (Academic, New York, 1971), p. 171. For a discussion of the experimental testability of his results, see Appendix B.

[5]J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969); J. F. Clauser, Bull. Am. Phys. Soc. 14, 578 (1969); A. Shimony, in *Foundations of Quantum Mechanics* (Ref. 4), p. 182; M. A. Horne, Ph.D. thesis, Boston University, 1970 (unpublished).

[6]S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. 28, 938 (1972); S. J. Freedman, Ph.D. thesis, University of California, Berkeley, 1972 [Lawrence Berkeley Laboratory Report No. LBL-391 (unpublished)]. See also Ref. 24.

[7]A. Shimony, in *Foundations of Quantum Mechanics* (Ref. 4), p. 191.

[8]We use the word "particle" in the conventional way to identify experimental phenomena exemplifying the general situation studied. Thus we do not assume here the existence of a microscopic entity with a "particle-like" structure.

[9]The practical criterion for "coincident counts" always involves a coincidence time window τ: Pairs of counts separated in time by less than τ are defined to be coincident. This procedure may appear to make the definition of $N_{12}(a,b)$ ambiguous since in general it will depend upon the experimenter's choice for τ. However, this dependence is usually insensitive to variations in τ which satisfy $s \ll \tau \ll 1/r$, where r is the average count rate at either detector, and s is the typical time separation of "true" coincidence pairs. Thus, we tacitly require the experimental arrangement to be such that this condition obtains (suitable source strength, time separation of pairs, etc.). If a sufficiently weak source is used, the ratio of "chance" coincident counts to "true" coincident counts can be made arbitrarily small, and the corresponding dead time can also be minimized.

[10]For the case where the emissions are spatially localized objects in flight from the source to each apparatus, such an intermediate time clearly exists. If the emissions are spatially extended systems (fields), which may take substantial time to exit the source, we consider the system after the emission process has commenced but before the leading edge has impinged on either apparatus.

[11]Even though we have introduced λ as the state of a specific *single* system, the assumed objectivity of the system described by this state allows us to consider an ensemble of these, physically identical to the extent that they are all characterized by the same λ. The probabilities are to be associated with this ensemble. Clearly, this procedure is conceptually sound, even in cases where we cannot in practice prepare the pure λ ensemble.

[12]Previous discussions (Refs. 1, 4, and 5) consider separately the passage of the emission through the analyzer and the detection of the emission after it has passed. Since "passage through analyzer" is not directly observable for microsystems, we consider as a unit the complete causal process from initial emission to final macroscopic effect (count). Moreover, the concept "passage through analyzer" is inappropriate for many field theories (classical, quantum-mechanical, or others) in which the emission may partially pass the analyzer.

[13]We should emphasize that an assumption is made here. By writing the density as $\rho(\lambda)$, instead of a more general conditional $\rho(\lambda|a,b)$, we deny such objective and local possibilities as these:

(a) Systems of some type originate the source which, reflecting off the analyzers and returning to the source, significantly effect the ensemble in a manner dependent upon a and b.

(b) Systems originate at the analyzers and impinge upon the source, thus effecting the ensemble in a manner dependent upon a and b.

(c) Systems originate within the intersection of the backward light cones of both analyzers and the source. These propagate into the spatial region of the whole apparatus, and simultaneously effect both the experimenters' selections of analyzer orientations and the emissions from the source.

In principle, (a) and (b) can be ruled out experimentally by rapidly and repeatedly changing the analyzer orientations immediately before each data-collection period and then stopping each data-collection period before the new orientations can be communicated (at the speed of light) to the source. The more general type-(c) conspiracies are difficult to rule out experimentally. But, for example, if the orientations a and b are selected at random by two physicists supposedly acting independently, significant cases of (c) require that their selections are *not* independent but are, in fact, strongly correlated with each other as well as with the source emissions.

[14]Because the form of λ is unspecified, we make no commitment to any specific field model, but adopt a quite general view that a field is an objective physical entity extended throughout space.

[15]Recall footnotes 10, 11, and 13 for additional informal justification of the name. The class of OLT includes the deterministic local hidden-variable theories discussed by Bell (Ref. 1) and Clauser *et al.* (Ref. 5) and is essentially the class of stochastic hidden-variable theories "with a certain local character" considered by Bell (Ref. 4). Hence we drop the name "hidden-variable theory," since traditionally the term has been used to identify theories characterized essentially by dispersion-free states, i.e., deterministic theories.

[16]Jauch [in *Foundations of Quantum Mechanics* (Ref. 4), p. 22] makes a similar conjecture when he suggests that "On the whole, and in spite of pretensions to the contrary, physicists are usually more inclined to Realism than to Positivism." However, for a specific example of implicit support for (2′), see M. L. Goldberger and K. M. Watson, Phys. Rev. 134, B919 (1964). Considering a system composed of two spin-$\frac{1}{2}$ particles, they comment: "In general therefore the observation of the orientation of spin 1 with respect to the axis has an instantaneous effect on the state of the second spin. With the interpretation of an observation as making a selection among the members of an ensemble, this is in no sense surprising." But a denial of any instantaneous effect for each member of this ensemble is tantamount to assuming (2′). The suprise, then, for an advocate of OLT, is that (2′) and quantum mechanics are incompatible.

[17]This identification, for the restricted case that each state of the mixture is a product of single-particle *quantum-mechanical* states, was conjectured by W. H. Furry [Phys. Rev. 49, 393 (1936); 49, 476 (1936)] and shown to be in disagreement with experiment by D. Bohm and Y. Aharonov [Phys. Rev. 108, 1070 (1957)]. An OLT is the generalization to mixtures of *any* type of product states.

[18]D. Bohm, *Quantum Theory* (Prentice-Hall, Englewood Cliffs, N. J., 1951), p. 614.

[19]C. S. Wu and I. Shaknov, Phys. Rev. 77, 136 (1950).

[20]L. Kasday, J. Ullman, and C. S. Wu, Bull. Am. Phys. Soc. 15, 586 (1970); see also L. Kasday, in *Foundations of Quantum Mechanics* (Ref. 4), p. 195.

[21]Actually, in atomic cascade experiments, the failure to violate (4′) is much worse than indicated in (6), since the singles probabilities p_1 and p_2 in (5) do not include the large contributions due to unpaired emissions from each transition of the cascade.

[22]The experiment of Kasday, Ullman, and Wu (Ref. 20) has confirmed the quantum-mechanical predictions for coincident Compton scattering of photon pairs produced in positronium annihilation. However, explicit OLT models exist (Refs. 5 and 20) which demonstrate that the analyzers (Compton polarimeter) are too inefficient to reveal a discrepancy between general OLT and quantum mechanics.

[23]Semiclassical radiation theories are OLT which satisfy the no-enhancement assumption. For an earlier discussion of these theories and two-photon correlations, see J. F. Clauser, Phys. Rev. A 6, 49 (1972). See also J. F. Clauser, Phys. Rev. D 9, 853 (1974).

[24]Experimental results in conflict with those of Freedman and Clauser have been found by R. A. Holt [Ph.D. thesis, Harvard University, 1973 (unpublished)]. He carried out the design of Ref. 5, as they did, but with a different atomic source of photon pairs and different optical arrangements. Holt's data agree with inequality (11′) and differ significantly from the predicted quantum-mechanical values. Further work is being pursued at various laboratories to explore this discrepancy.

[25]The upper bound on c_2, necessary to keep $p_2(\lambda, b)$ sensible, depends on the analyzer efficiencies ϵ_M^2 and ϵ_m^2 (through the functions ϵ_+^2, ϵ_-^2, and δ) and is independent of the detector efficiency η_2. On the other hand, a physical interpretation of Eq. (16) suggests that $c_2 = \eta_2$. Then if both the detector efficiencies are larger than 0.38, some probability of the model will not be sensible. However, there are undoubtedly OLT models that remain sensible for substantially larger efficiencies. Moreover, the identification $c_2 = \eta_2$ is not a logical necessity. The quantum efficiency definition for a photomultiplier depends on a long chain of experimentation and accepted theory. Recall the related comments concerning N in Sec. III.

[26]A qualitative argument with conclusions similar to ours has been given by B. d'Espagnat, in *Proceedings of the 1972 Trieste Conference on the Physicist's Conception of Nature*, edited by J. Mehra (Reidel, Dordrecht, Holland, 1973). See also B. d'Espagnat, *Conceptional Foundations of Quantum Mechanics* (Benjamin, Menlo Park, California, 1971), Chaps. 8, 9, and 16.

[27]Bell actually considers spin-$\frac{1}{2}$ pairs; we give the photon analogy.

[28]Bell does not write Eqs. (B1), but this definition is implicit in his discussion.

[29]If the no-count outcomes (denoted 0) are considered as well as the + and − ones, there are nine sets of inequalities. But only the four given in (B3) are of immediate experimental utility, since the other five all contain probabilities of nothing happening.