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Experimental Test of Local Hidden-Variable Theories*

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We have measured the linear polarization correlation of the photons emitted in an atomic cascade of calcium. It has been shown by a generalization of Bell's inequality that the existence of local hidden variables imposes restrictions on this correlation in conflict with the predictions of quantum mechanics. Our data, in agreement with quantum mechanics, violate these restrictions to high statistical accuracy, thus providing strong evidence against local hidden-variable theories.

Since quantum mechanics was first developed, there have been repeated suggestions that its statistical features possibly might be described by an underlying deterministic substructure. Such

features, then, arise because a quantum state represents a statistical ensemble of "hidden-variable states." Proofs by von Neumann and others, demonstrating the impossibility of a hid-

den-variable substructure consistent with quantum mechanics, rely on various assumptions concerning the character of the hidden variables.¹ Bell has argued that these assumptions are unduly restrictive. However, by considering an idealized case of two spatially separated but quantum-mechanically correlated systems, he was able to show that any hidden-variable theory satisfying only the natural assumption of "locality" also leads to predictions ("Bell's inequality") in conflict with quantum mechanics.²

Bell's proof was extended to realizable systems by Clauser, Horne, Shimony, and Holt,³ who also pointed out that their generalization of Bell's inequality can be tested experimentally, thus testing all local hidden-variable theories, but that existing experimental results were insufficient for this purpose. This Letter reports the results of an experiment which are sufficiently precise to rule out local hidden-variable theories with high statistical accuracy

In the present work we measured the correlation in linear polarization of two photons γ_1 and γ_2 emitted in a $J=0 - J=1 - J=0$ atomic cascade. The decaying atoms were viewed by two symmetrically placed optical systems, each consisting of two lenses, a wavelength filter, a rotatable and removable polarizer, and a single-photon detector (see Fig. 1). The following quantities were measured: $R(\varphi)$, the coincidence rate for two-photon detection, as a function of the angle φ between the planes of linear polarization defined by the orientation of the inserted polarizers; R_1 , the coincidence rate with polarizer 2 removed; R_2 , the coincidence rate with polarizer 1 removed; R_0 , the coincidence rate with both polarizers re-

moved. Quantum mechanics predicts that $R(\varphi)$ and R_0 are related as follows^{3,5}:

$$R(\varphi)/R_0 = \frac{1}{4}(\epsilon_M^1 + \epsilon_m^1)(\epsilon_M^2 + \epsilon_m^2) + \frac{1}{4}(\epsilon_M^1 - \epsilon_m^1) \times (\epsilon_M^2 - \epsilon_m^2)F_1(\theta) \cos 2\varphi, \quad (1a)$$

while

$$R_1/R_0 = \frac{1}{2}(\epsilon_M^1 + \epsilon_m^1), \quad (1b)$$

and

$$R_2/R_0 = \frac{1}{2}(\epsilon_M^2 + \epsilon_m^2). \quad (1c)$$

Here ϵ_M^i (ϵ_m^i) is the transmittance of the i th polarizer for light polarized parallel (perpendicular) to the polarizer axis, and $F_1(\theta)$ is a function of the half-angle θ subtended by the primary lenses. It represents a depolarization due to noncollinearity of the two photons, and approaches unity for infinitesimal detector solid angles. [For this experiment, $\theta \approx 30^\circ$, and $F_1(30^\circ) = 0.99$.]

We make the following assumptions for any local hidden-variable theory: (1) The two photons propagate as separated localized particles. (2) A binary selection process occurs for each photon at each polarizer (transmission or no-transmission). This selection does not depend upon the orientation of the distant polarizer.

In addition, we make the following assumption to allow a comparison of the generalization of Bell's inequality with our experiment: (3) All photons incident on a detector have a probability of detection that is independent of whether or not the photon has passed through a polarizer.⁶

The above assumptions constrain the coincidence rates by the following inequalities⁷:

$$-1 \leq \Delta(\varphi) \leq 0, \quad (2)$$

where

$$\Delta(\varphi) = \frac{3R(\varphi)}{R_0} - \frac{R(3\varphi)}{R_0} - \frac{R_1 + R_2}{R_0}.$$

For sufficiently small detector solid angles and highly efficient polarizers, these inequalities (2) are not satisfied by the quantum-mechanical prediction (1) for a range of values of φ . Maximum violations occur at $\varphi = 22\frac{1}{2}^\circ$ [$\Delta(\varphi) > 0$] and $\varphi = 67\frac{1}{2}^\circ$ [$\Delta(\varphi) < -1$]. At these angles of maximum violation, inequalities (2) can be combined into the simpler and more convenient expression

$$\delta = |R(22\frac{1}{2}^\circ)/R_0 - R(67\frac{1}{2}^\circ)/R_0| - \frac{1}{4} \leq 0, \quad (3)$$

which does not involve R_1 or R_2 .

The experimental arrangement was similar to

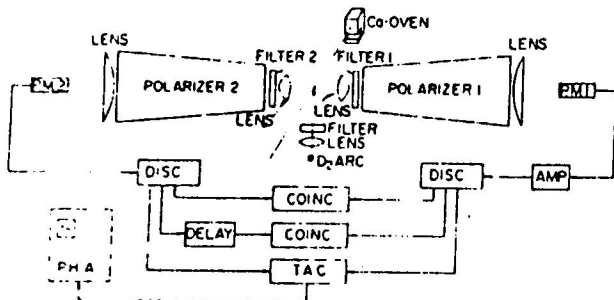


FIG. 1. Schematic diagram of apparatus and associated electronics. Scalars (not shown) monitored the outputs of the discriminators and coincidence circuits during each 100-sec count period. The contents of the scalars and the experimental configuration were recorded on paper tape and analyzed on an IBM 1620-II computer.

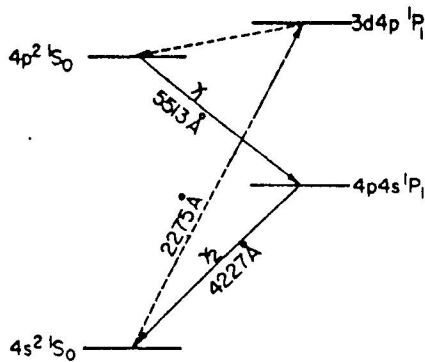


FIG. 2. Level scheme of calcium. Dashed lines show the route for excitation to the initial state $4p^2 1S_0$.

that of Kocher and Commins.⁸ A calcium atomic beam effused from a tantalum oven, as shown in Fig. 1. The continuum output of a deuterium arc lamp (ORIEL C-42-72-12) was passed through an interference filter [250 Å full width at half-maximum (FWHM), 20% transmission at 2275 Å] and focused on the beam. Resonance absorption of a 2275-Å photon excited calcium atoms to the $3d4p 1P_1$ state. Of the atoms that did not decay directly to the ground state, about 7% decayed to the $4p^2 1S_0$ state, from which they cascaded through the $4s4p 1P_1$ intermediate state to the ground state with the emission of two photons at 5513 Å (γ_1) and 4227 Å (γ_2) (see Fig. 2). At the interaction region (roughly, a cylinder 5 mm high and 3 mm in diameter) the density of the calcium was about 1×10^{10} atoms/cm³. To avoid spherical aberrations which would have reduced counter efficiencies, aspheric primary lenses (8.0 cm diam, $f=0.8$) were used. Photons γ_1 were selected by a filter with 10 Å FWHM and 50% transmission, and γ_2 by a filter with 6 Å FWHM and 20% transmission. The requirement for large efficient linear polarizers led us to employ "pile-of-plates" polarizers. Each polarizer consisted of ten 0.3-mm-thick glass sheets inclined nearly at Brewster's angle. The sheets were attached to hinged frames, and could be folded completely out of the optical path. A Geneva mechanism rotated each polarizer through increments of $22\frac{1}{2}^\circ$. The measured transmittances of the polarizers were $\epsilon_M^1 = 0.97 \pm 0.01$, $\epsilon_m^1 = 0.038 \pm 0.004$, $\epsilon_M^2 = 0.96 \pm 0.01$, and $\epsilon_m^2 = 0.037 \pm 0.004$. The photomultiplier detectors (RCA C31000E, quantum efficiency ≈ 0.13 at 5513 Å; and RCA 8850, quantum efficiency ≈ 0.28 at 4227 Å) were cooled, reducing dark rates to 75 and 200 counts/sec, respectively. The measured counter efficiencies with po-

larizers removed were $\eta_1 \approx 1.7 \times 10^{-8}$ and $\eta_2 \approx 1.5 \times 10^{-8}$.

A diagram of the electronics is included in Fig. 1. The overall system time resolution was about 1.5 nsec. The short intermediate state lifetime (~ 5 nsec) permitted a narrow coincidence window (8.1 nsec). A second coincidence channel displaced in time by 50 nsec monitored the number of accidental coincidences, the true coincidence rate being determined by subtraction.¹⁰ A time-to-amplitude converter and pulse-height analyzer measured the time-delay spectrum of the two photons. The resulting exponential gave the intermediate state lifetime.¹¹

The coincidence rates depended upon the beam and lamp intensities, the latter gradually decreasing during a run. The typical coincidence rate with polarizers removed ranged from 0.3 to 0.1 counts/sec, and the accidental rate ranged from 0.01 to 0.002 counts/sec. Long runs required by the low coincidence rate necessitated automatic data collections.

The system was cycled with 100-sec counting periods. Periods with one or both polarizers inserted alternated with periods in which both polarizers were removed. Both polarizers rotated according to a prescribed sequence. For a given run, $R(\varphi)/R_0$ was calculated by summing counts for all configurations corresponding to angle φ and dividing by half the sum of the counts in the adjacent periods of the sequence in which both polarizers were removed. Data for R_1/R_0 and R_2/R_0 were analyzed in a similar fashion. The values given here are averages over the orientation of the inserted polarizer. This cycling and averaging procedure minimized the effects of drift and apparatus asymmetry.

The results of the measurements of the correlation $R(\varphi)/R_0$, corresponding to a total integration time of ~ 200 h, are shown in Fig. 3. All error limits are conservative estimates of 1 standard deviation. Using the values at $22\frac{1}{2}^\circ$ and $67\frac{1}{2}^\circ$, we obtain $\delta = 0.050 \pm 0.008$ in clear violation of inequality (3).¹² Furthermore, we observe no evidence for a deviation from the predictions of quantum mechanics, calculated from the measured polarizer efficiencies and solid angles, and shown as the solid curve in Fig. 3. We consider these results to be strong evidence against local hidden-variable theories.

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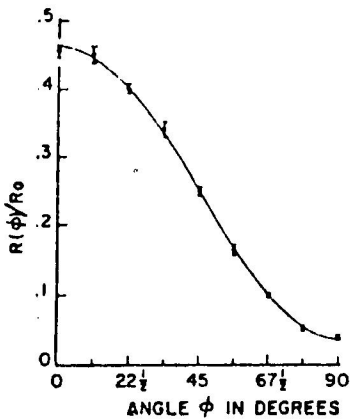


FIG. 3. Coincidence rate with angle ψ between the polarizers, divided by the rate with both polarizers removed, plotted versus the angle ψ . The solid line is the prediction by quantum mechanics, calculated using the measured efficiencies of the polarizers and solid angles of the experiment.

mons for helpful suggestions.

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¹The best-known proof is by J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer, Berlin, 1932) [*Mathematical Foundations of Quantum Mechanics* (Princeton Univ. Press, Princeton, N. J., 1955)]. For a critical review of this and other proofs see J. S. Bell, *Rev. Mod. Phys.* **38**, 447 (1966).

²J. S. Bell, *Physics* (Long Is. City, N.Y.) **1**, 195 (1964).

³J. Clauser, M. Horne, A. Shimony, and R. Holt, *Phys. Rev. Lett.* **23**, 880 (1969).

⁴A hidden-variable theory need not require that R_1 and R_2 be independent of the orientation of the inserted polarizer, and we do not assume this independence in our data analysis. However, the results are consistent with R_1 and R_2 being independent of angle, and for simplicity they are so denoted.

⁵M. Horne, Ph. D. thesis, Boston University, 1970 (unpublished). See also A. Shimony, in "Foundations of Quantum Mechanics, Proceedings of the International School of Physics 'Enrico Fermi,' Course IL" (Academic, New York, to be published).

⁶This assumption is much weaker than the assumption made by L. R. Kasday, J. Ullman, and C. S. Wu, *Bull. Amer. Phys. Soc.* **15**, 586 (1970), in their discussion of the two- γ decay of positronium; see L. R. Kasday, in "Foundations of Quantum Mechanics, Proceedings of the International School of Physics 'Enrico Fermi,' Course IL" (Academic, New York, to be published).

⁷The inequality $\Delta(\psi) \leq 0$ is derived in Refs. 3 and 5. The other forms of the hidden-variable restriction are obtained by similar arguments; see S. Freedman, Ph. D. thesis, University of California, Berkeley, Lawrence Berkeley Laboratory Report No. LBL-391, 1972 (unpublished).

⁸C. A. Kocher and E. D. Commins, *Phys. Rev. Lett.* **18**, 575 (1967); C. A. Kocher, Ph. D. thesis, University of California, Berkeley, Lawrence Berkeley Laboratory Report No. UCRL-17587, 1967 (unpublished).

⁹The counter efficiencies are given by $\eta_i = (\Omega_i/4\pi)T_i \times \epsilon_i L_i$, where Ω_i is the solid angle, T_i is the transmission of the filter, ϵ_i is the quantum efficiency, and L_i accounts for other losses. The measurement of η_2 was made, employing the properties of the calcium cascade, by comparing the coincidence rate and the γ_1 singles rate after suitable background correction; η_1 was then inferred from the known quantum efficiencies and filter transmissions assuming that Ω_i and L_i were the same for both detector systems.

¹⁰An estimate of the accidental rate was also obtained from the singles rates. The two estimates gave consistent results. In fact, our conclusions are not changed if accidentals are neglected entirely; the signal-to-accidental ratio with polarizer removed is about 40 to 1 for the data presented.

¹¹Resonance trapping, encountered at high beam densities, resulted in a lengthening of the observed lifetime and a slight decrease in the polarization correlation amplitude, see J. P. Barrat, *J. Phys. Radium* **20**, 541, 633 (1959). At low beam densities the measured lifetime is consistent with previously measured values. See W. L. Weise, M. W. Smith, and B. M. Miles, *Atomic Transition Probabilities*, U. S. National Bureau of Standards Reference Data Series—22 (U.S. GPO, Washington, D.C., 1969), Vol. 2.

¹²The results that are of interest in comparison with the hidden-variable inequalities are $R_1/R_0 = 0.497 \pm 0.009$, $R_2/R_0 = 0.499 \pm 0.009$, $R(22\frac{1}{2}^\circ)/R_0 = 0.400 \pm 0.007$, and $R(67\frac{1}{2}^\circ)/R_0 = 0.100 \pm 0.003$. We thus obtain $\Delta(22\frac{1}{2}^\circ) = 0.104 \pm 0.026$ and $\Delta(67\frac{1}{2}^\circ) = -1.097 \pm 0.018$, in violation of inequalities (2).